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# Modes of a laser resonator with a retroreflecting corner cube mirror

Guosheng Zhou, Anthony J. Alfrey, and Lee W. Casperson

The self-consistent integral equation for the field distribution of the resonant modes in a resonator with a tilted retroreflecting corner cube mirror is solved. The corner cube acts like a convex lens with radius of curvature  $-L \cot^2 \theta$  in the rotation direction ( $L$  is the cavity length and  $\theta$  the rotation angle) and like a flat plane in the direction of the rotation axis. The field distribution can be described in terms of Hermite-Gaussian functions, and these results have been confirmed experimentally using an Ar-ion laser. The equivalent beam matrix for a reflecting corner cube is also found.

## I. Introduction

It was recently pointed out that the modes of a laser resonator with a retroreflecting roof mirror tilted in the direction of the roof are almost the same as the familiar Hermite-Gaussian field distributions.<sup>1</sup> The resulting resonator is less sensitive to misalignment in one special direction. The natural development of this concept is to look for a new type of resonator which is less sensitive to misalignment in any direction. Laser resonators with retroreflective corner cube reflectors are found to meet this need. They might also be useful for compensating prismlike distortions in chemical lasers or large volume gas lasers.

The polarization properties of a laser resonator with a pair of corner cube reflectors or a corner cube and a flat mirror were discussed by Peck.<sup>2</sup> The modes in these resonators, however, have not been investigated previously. This paper presents an analysis of the modes of a resonator with a corner cube reflector having arbitrary angular alignment. The modes of a resonator consisting of a corner cube reflector facing a spherical mirror are found to be almost the same as the Hermite-Gaussian distributions obtained in conventional spherical mirror resonators.

The basic self-consistent integral equation is derived in Sec. II, and the solutions for resonators with corner cube and spherical mirrors are given in Sec. III. Our experimental results obtained with Ar-ion corner cube lasers are described in Sec. IV.

## II. Self-Consistent Integral Equation

The retroreflecting corner cube resonator is schematically represented in Fig. 1. It consists of a tilted corner cube reflector facing a spherical mirror. The

reflectivities of both corner cube and spherical mirrors are assumed to be uniform. The spherical mirror has a radius of curvature  $R_2$ . Geometrical properties of the corner cube are considered first. For simplicity, the coordinate system is chosen to be coincident with the corner cube. That is, the coordinate origin is at the vertex of the corner, and the  $\xi$ ,  $\eta$ , and  $\zeta$  axes are coincident with each of the three roofs of the corner cube (see Fig. 2). However, most of our results are in vector forms which are independent of the coordinate system.

A ray which is incident on the front surface of the corner cube  $ABC$  at the point  $P_i(\xi_i, \eta_i, \zeta_i)$  in an arbitrary direction described by  $S_i(l, m, n)$  ( $l$ ,  $m$ , and  $n$  are the direction cosines) can be expressed in the form

$$(\mathbf{r} - \mathbf{r}_i) \times \mathbf{S}_i = 0, \quad (1)$$

where  $\mathbf{r} = (\xi, \eta, \zeta)$  is the variable vector, while  $\mathbf{r}_i = (\xi_i, \eta_i, \zeta_i)$  is the vector from the origin to the point of incidence. The ray then impinges on one of the three reflecting surfaces of the corner cube, for example, plane  $AOC$ , which can be expressed by

$$\eta = 0. \quad (2)$$

From Eqs. (1) and (2) one can find the intersection  $A'_1$  of the incident ray and the plane expressed by Eq. (2),

$$\mathbf{r}_1 = \left( \xi_i - \frac{l}{m} \eta_i, 0, \zeta_i - \frac{n}{m} \eta_i \right), \quad (3)$$

where  $\mathbf{r}_1$  is the position vector of the point  $A'_1$ .

The ray given by Eq. (1) then experiences a reflection at the point  $A'_1$ . The direction of the reflected ray  $\mathbf{S}_1$  satisfies the following equations according to Snell's law:

$$(\mathbf{S}_1 + \mathbf{S}_i) \cdot \hat{\mathbf{j}} = 0 \quad (\mathbf{S}_1 - \mathbf{S}_i) \times \hat{\mathbf{j}} = 0 \quad (4)$$

where  $\hat{\mathbf{j}} = (0, 1, 0)$  is the unit vector normal to surface (1). Solving Eqs. (4) one obtains

$$\mathbf{S}_1 = (l, -m, n). \quad (5)$$

Then the ray reflected from surface (2) can be written in the form

$$(\mathbf{r} - \mathbf{r}_1) \times \mathbf{S}_1 = 0. \quad (6)$$

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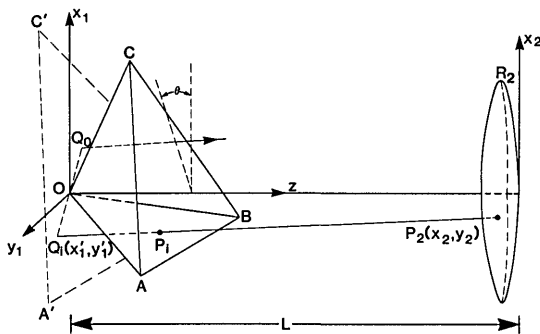


Fig. 1. Schematic representation of a resonator with a corner cube and spherical mirror.

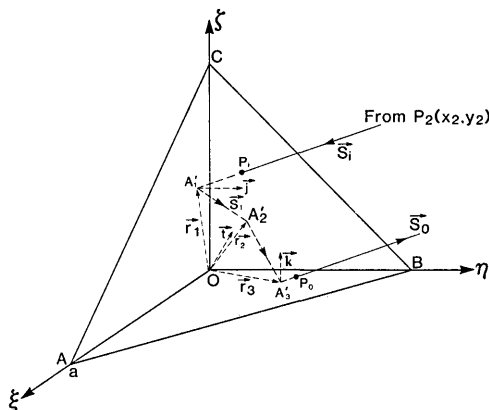


Fig. 2. Schematic representation of a light ray in the retroreflecting corner cube.

In the same way one can get the intersections of the ray with the other two surfaces of the corner cube. They are

$$\mathbf{r}_2 = (0, -\eta_i + m\xi_i/l, \zeta_i - \eta\xi_i/l), \quad (7)$$

$$\mathbf{r}_3 = (-\xi_i + l\zeta_i/n, -\eta_i + m\zeta_i/n, 0). \quad (8)$$

After three reflections on the three reflecting surfaces of the corner cube the ray is parallel to the incident ray and meets the front surface ABC of the corner cube at point  $P_0(\xi_0, \eta_0, \zeta_0)$  whose position vector  $\mathbf{r}_0$  is

$$\mathbf{r}_0 = -\mathbf{r}_i + 2at\mathbf{S}_i/(\mathbf{S}_i \cdot \mathbf{t}), \quad (9)$$

where  $a = OA = OB = OC$  is the length of the roof as shown in Fig. 2,  $\mathbf{t}$  is the vector which is perpendicular to the surface ABC and starts from the vertex of the corner cube O, and  $t$  is the length of the vector  $\mathbf{t}$  ( $t = |\mathbf{t}|$ ). From Eqs. (3), (7), (8), and (9) one finds that the path length difference  $\Delta$  from the point  $P_i$  through  $A_1$ ,  $A_2$ , and  $A_3$  to  $P_0$  is

$$\Delta = P_iA_1 + A_1A_2 + A_2A_3 + A_3P_0 = 2at/(\mathbf{S}_i \cdot \mathbf{t}). \quad (10)$$

One may now imagine a plane  $A'B'C'$  which passes through the vertex O and is parallel to the surface ABC (see Fig. 3). This plane can be expressed as

$$\xi + \eta + \zeta = 0. \quad (11)$$

If the incident ray  $P_iA_1$  and the outgoing ray  $A_3P_0$  are extended, they intersect the plane  $A'B'C'$  at the points  $Q_i$  and  $Q_0$ , respectively. From Eqs. (9) and (11) one finds

$$\mathbf{Q}_i = \left( \xi_i - \frac{la}{l+m+n}, \eta_i - \frac{ma}{l+m+n}, \zeta_i - \frac{na}{l+m+n} \right) \quad (12)$$

$$\mathbf{Q}_0 = -\mathbf{Q}_i, \quad (13)$$

where  $\mathbf{Q}_i$  and  $\mathbf{Q}_0$  are the vectors which start from the vertex O and go to the points  $Q_i$  and  $Q_0$ , respectively. Thus, on the surface  $A'B'C'$  the incident and outgoing points  $Q_i$  and  $Q_0$  are symmetric to the vertex O.

The path length difference  $\Delta' \equiv P_iQ_i + P_0Q_0$  can be obtained from Eqs. (12) and (13),

$$\Delta' = 2at/(\mathbf{S}_i \cdot \mathbf{t}) = \Delta. \quad (14)$$

Consequently, a ray from  $P_i$  through  $A_1$ ,  $A_2$ , and  $A_3$  to  $P_0$  can be equivalent to a ray which propagates from  $P_i$  through  $A_1$  to  $Q_i$ , experiences a displacement  $Q_iQ_0$  which does not contribute to any path length difference, and then travels from  $Q_i$  through  $A_3$  to  $P_0$ . Therefore, the plane  $A'B'C'$  can be regarded as a reference plane for the calculation of fields.

For the sake of convenience and simplicity, we now choose the coordinate system as shown in Fig. 1. The coordinate origin is the vertex of the corner cube, and the distance between the center of the spherical mirror and the vertex 0 is represented by  $L$ . The optical axis ( $z$  axis) passes through the center of the spherical mirror and the vertex of the corner cube O. The  $y_1$  axis is coincident with the axis of rotation of the corner cube. The angle between the  $x_1$  axis and the surface of the corner cube is  $\theta$ , which is positive when the surface of the corner cube is turned counterclockwise from the  $x_1$  axis. The points  $Q_i$  and  $Q_0$  can be represented in this coordinate system by  $Q_i(x'_1, y'_1)$  and  $Q_0(x_1, y_1)$ , respectively, and according to their symmetry property Eq. (13), we have

$$x'_1 = -x_1, \quad y'_1 = -y_1. \quad (15)$$

The path length difference from the point  $P_2(x_2, y_2)$  through  $P_i$  to  $Q_i(x'_1, y'_1)$  can be expressed in the form

$$\rho(x'_1, y'_1; x_2, y_2) = \rho(x'_1, x_2) + \rho'(y'_1, y_2), \quad (16)$$

where

$$\rho(x'_1, x_2) \simeq L/2 + x'_1 \tan \theta + (1/2L)(x_1^2 \sec^2 \theta + g_2 x_2^2 - 2x'_1 x_2), \quad (17)$$

$$\rho'(y'_1, y_2) = L/2 + (1/2L)(y_1^2 + g_2 y_2^2 - 2y'_1 y_2), \quad (18)$$

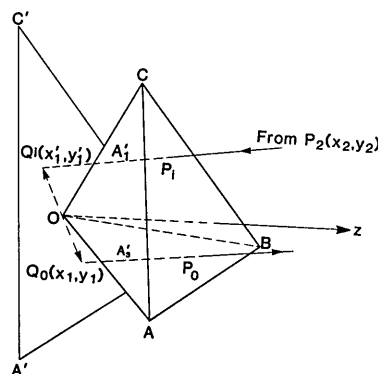


Fig. 3. Schematic representation of the imaginary reference plane  $A'B'C'$ .

and  $g_2 = 1 - L/R_2$ . By substituting Eq. (15), Eqs. (17) and (18) turn out to be

$$\begin{aligned}\rho(x_1', x_2) &= L/2 - x_1 \tan\theta \\ &+ (1/2L)(x_1'^2 \sec^2\theta + g_2 x_2^2 + 2x_1 x_2) \\ &\equiv \rho_x(x_1, x_2),\end{aligned}\quad (19)$$

$$\begin{aligned}\rho(y_1', y_2) &= L/2 + (1/2L)(y_1'^2 + g_2 y_2^2 + 2y_1 y_2) \\ &\equiv \rho_y(y_1, y_2).\end{aligned}\quad (20)$$

The path length difference from the point  $Q_0(x_1, y_1)$  to  $P_2(x_2, y_2)$  is given by

$$\rho(x_1, y_1; x_2, y_2) = \rho(x_1, x_2) + \rho(y_1, y_2). \quad (21)$$

The self-consistent Fresnel-Kirchhoff integral equation can be expressed as follows:

$$\gamma E^{(2)}(x_2, y_2) = \iint dx_2' dy_2' K(x_2, y_2; x_2', y_2') E^{(2)}(x_2', y_2'), \quad (22)$$

$$\begin{aligned}K(x_2, y_2; x_2', y_2') &= (i/\lambda)^2 \iint dx_1' dy_1' \\ &\times [\rho(-x_1', -y_1'; x_2, y_2) \rho(x_1', y_1'; x_2', y_2')]^{-1} \\ &\times \exp[-ik\rho(-x_1', -y_1'; x_2, y_2)] \\ &\times \exp[-ik\rho(-x_1', -y_1'; x_2', y_2)] \\ &- ik\rho(x_1, y_1; x_2, y_2)],\end{aligned}\quad (23)$$

where the double primes denote integration variables. The field on the surface of the spherical mirror is represented by  $E^{(2)}(x_2, y_2)$ , and  $\gamma$  is the corresponding eigenvalue.  $K$  is the kernel of the integration equation. The wave number is  $k = 2\pi/\lambda$ , and  $\lambda$  is the wavelength in the resonator.

Since the kernel of the integral equation  $K(x_2, y_2; x_2', y_2')$  can be separated in the  $x$  and  $y$  directions, the field expression splits into two independent integral equations, one for each lateral coordinate. These can be written as follows:

$$\begin{aligned}\gamma_x E_x^{(2)}(x_2) &= (i/\lambda L) \int dx_2' E_x^{(2)}(x_2') \int dx_1' (1 - x_1' \tan\theta/L)^{-1} \\ &\times (1 + x_1' \tan\theta/L)^{-1} \exp[-ik\{\rho_x(x_1', x_2) \\ &+ \rho(x_1', x_2)\}],\end{aligned}\quad (24)$$

$$\begin{aligned}\gamma_y E_y^{(2)}(y_2) &= (i/\lambda L) \int dy_2' E_y^{(2)}(y_2') \int dy_1' \\ &\times \exp[-ik\{\rho_y(y_1', y_2) - ik\rho'(y_1', y_2)\}],\end{aligned}\quad (25)$$

$$E^{(2)}(x_2, y_2) = E_x^{(2)}(x_2) E_y^{(2)}(y_2), \quad (26)$$

$$\gamma = \gamma_x \gamma_y, \quad (27)$$

where  $E_x^{(2)}(x_2)$  and  $E_y^{(2)}(y_2)$  illustrate, respectively, the field variation in the  $x$  and  $y$  directions on the surface of the spherical mirror with  $\gamma_x$  and  $\gamma_y$  as the corresponding eigenvalues. According to the approximate rules used in the Huygens-Kirchhoff equation, in the integrand function small quantities to the first order are kept and in the exponential small quantities to the second order are kept. The factor  $(1 - x_1' \tan\theta/L)(1 + x_1' \tan\theta/L)$  comes from the distance  $P_2 Q_i$  and  $Q_0 P_2$  in the denominator. The limits of the second integration with respect to  $x_1'$  and  $y_1'$  are assumed to approach infinity, which can be interpreted to mean that the modes of the resonator will be sufficiently confined about the axis and the corner cube be sufficiently large so contributions to the integration from points other than those close to axis may be neglected.

By substituting Eqs. (17) to (20) into Eqs. (24) and (25) and integrating with respect to  $x_1'$  and  $y_1'$  we can simplify Eqs. (24) and (25) as follows:

$$\begin{aligned}\gamma_x E_x^{(2)}(x_2) &= \exp(i\pi/4 - ikL)(2\lambda L g_x)^{-1/2} \int_{-\infty}^{\infty} dx_2' E_x^{(2)}(x_2') \\ &\times \exp\{(-ik/4L)[(2g_x g_2 - 1)(x_2'^2 + x_2^2) + 2x_2 x_2']\},\end{aligned}\quad (28)$$

$$\begin{aligned}\gamma_y E_y^{(2)}(y_2) &= \exp(i\pi/4 - ikL)(2\lambda L)^{-1/2} \int_{-\infty}^{\infty} dy_2' E_y^{(2)}(y_2') \\ &\times \exp\{(-ik/4L)[(2g_2 - 1)(y_2'^2 + y_2^2) + 2y_2 y_2']\},\end{aligned}\quad (29)$$

where

$$g_x = \sec^2\theta. \quad (30)$$

### III. Modes of a Resonator With Corner Cube and Spherical Mirror

Equation (29) has been solved in Ref. 1. The field variation in the  $y$  direction can be expressed in the form

$$E_{yn}^{(2)} = \phi_n(y_2/w_2), \quad (31)$$

where

$$\phi_n(y_2/w_2) = N_n^{(2)} H_n(2^{1/2} y_2/w_2) \exp(-y_2^2/w_2^2), \quad (32)$$

$$N_n^{(2)} = (2^{1/2}/w_2)^{1/2} (2^n n! \pi^{1/2})^{-1/2}, \quad (33)$$

$$w_2 = (\lambda L/\pi)^{1/2} [(1 - g_2)g_2]^{-1/4}. \quad (34)$$

Here  $\phi_n$  is the normalized Hermite-Gaussian function,  $H_n$  is a Hermite polynomial of order  $n$ ,  $N_n^{(2)}$  is a normalization factor, and  $w_2$  is a beam spot size in the  $y$  direction on the spherical mirror. The corresponding eigenvalue can be expressed as

$$\gamma_{yn} = (-1)^n \sigma_n, \quad (35)$$

where

$$\begin{aligned}\sigma_n &= \exp[-ikL + i(n + 1/2) \\ &\times (\pi/2 + \tan^{-1}\{(1 - 2g_2)[1 - (1 - 2g_2)^2]^{-1/2}\})].\end{aligned}\quad (36)$$

By using the same procedure, one can get the solution of Eq. (28) as follows:

$$E_{xm}^{(2)} = \phi_m(x_2/w_2'), \quad (37)$$

where  $\phi_m$  is similar to Eq. (32), and the spot size in the  $x$  direction on the spherical mirror is

$$w_2' = (\lambda L/\pi)^{1/2} [(1 - g_x g_2)g_2/g_x]^{-1/4}. \quad (38)$$

The corresponding eigenvalue  $\gamma_{xm}$  is

$$\gamma_{xm} = (-1)^m \sigma_m', \quad (39)$$

where

$$\begin{aligned}\sigma_m' &= \exp[-ikL + i(m + 1/2)(\pi/2 + \tan^{-1}\{(1 - 2g_x g_2) \\ &\times [1 - (1 - 2g_x g_2)^2]^{-1/2}\})].\end{aligned}\quad (40)$$

It is interesting that even for large inclination angles of the corner cube in arbitrary directions, the field can still be expressed in terms of ordinary Hermite-Gaussian functions, and the propagation constant  $\gamma$  is the same as for a common spherical resonator except for a multiplier  $(-1)^{m+n}$ . This multiplier means that the odd symmetrical modes have an additional phase shift of  $\pi$ . These results are consequences of the retroreflecting property of the corner cube.

When the inclination angle is large, the field distribution appears a little bit astigmatic. The beam spot sizes in the  $x$  and  $y$  directions are slightly different. In the direction perpendicular to the inclination ( $y$  axis),

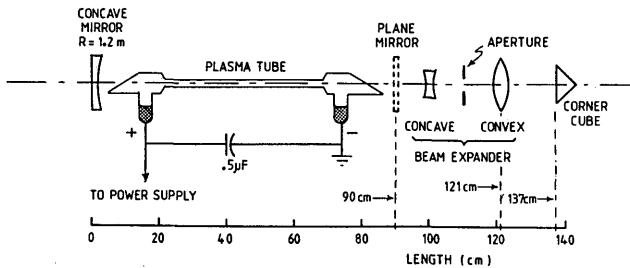


Fig. 4. Experimental setup for an Ar-ion corner cube laser.

the corner cube acts like a flat plane; while in the inclination direction ( $x$  axis), the corner cube acts like a convex lens with a variable radius of curvature

$$R = -L \cot^2 \theta. \quad (41)$$

Nevertheless, when the inclination is not very large and  $\tan^2 \theta \ll 1$ ,  $g_x \approx 1$ . Then the corner cube appears like a flat plane both in the  $x$  and  $y$  directions, and the field distribution is essentially nonastigmatic. The resonant frequency  $\nu_{mnq}$  can be found from Eqs. (27), (35), and (39);

$$\nu_{mnq} = (c/2L) \left[ q - \frac{m+n}{2} + (m + \frac{1}{2})\pi^{-1} \cos^{-1}(g_x g_2)^{1/2} + (n + \frac{1}{2})\pi^{-1} \cos^{-1} g_2^{1/2} \right]. \quad (42)$$

After the field distribution on the spherical mirror has been determined, the field variation in the  $x$  direction on the imaginary plane  $A'B'C'$  of the corner cube can be derived by Fresnel integration

$$E_{xm}^{(1)} = \chi_m^{(1)} (1 - x_1 \tan \theta / L) \exp(-ikx_1 \tan \theta) \phi_m^{(1)}(x_1/w_1'), \quad (43)$$

where

$$\chi_m' = \exp(-ikL/2 + i(m + \frac{1}{2}) \times \{\pi/2 - \tan^{-1}[g_x g_2 / (1 - g_x g_2)^{1/2}]\}), \quad (44)$$

$$\phi_m^{(1)}(x_1/w_1') = N_m^{(1)} H_m(2^{1/2} x_1/w_1') \exp(-x_1^2/w_1'^2), \quad (45)$$

$$N_m^{(1)} = (2^{1/2}/w_1')^{1/2} (2^m m! \pi^{1/2})^{-1/2}, \quad (46)$$

$$w_1' = (\lambda L / \pi)^{1/2} [(1 - g_x g_2) g_x / g_2]^{-1/4}.$$

Here the phase factor  $\exp(-ikx_1 \tan \theta)$  comes from the inclination of the corner cube, and  $(1 - x_1 \tan \theta / L)$  results from the denominator in the integrand.

The field variation in the  $y$  direction on the imaginary plane  $A'B'C'$  of the corner cube can be expressed in the form

$$E_{yn}^{(1)} = \chi_n \phi_n^{(1)}(y_1/w_1), \quad (47)$$

where  $\chi_n$  and  $\phi_n^{(1)}$  are in the same form as those in the  $x$  direction except that  $w_1'$  and  $g_x$  are replaced by  $w_1$  and 1, respectively, where

$$w_1 = (\lambda L / \pi)^{1/2} [(1 - g_2) g_2^{-1}]^{-1/2}. \quad (48)$$

When the inclination angle is small,  $\tan^2 \theta \ll 1$ , the beam matrix for the corner cube can be represented by

$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \quad (49)$$

which has been proved previously for one direction in a retroreflecting roof resonator.<sup>1</sup> When substituted into the appropriate phase transformation,<sup>3,4</sup> this matrix leads to the  $(-1)^{m+n}$  factor mentioned here.

The resonator containing a corner cube and a spherical mirror has some obvious advantages. First, the corner cube does not need alignment in any direction except perhaps to keep the optical axis in line with the amplifying medium and to maintain total internal reflection of the rays. Second, when the inclination angle  $\theta$  is small, the field distribution on the spherical mirror is just the same as the ordinary Hermite-Gaussian modes. Third, when the inclination angle  $\theta$  is large, the effective radius of curvature of the corner cube may be changed by rotation of the corner cube, and this effect might be used to compensate for astigmatism.

It should be noted that in the preceding analysis the fields have been assumed to be linearly polarized. In fact the polarization properties of light reflected from a corner cube may sometimes be more complex. However, certain polarization states may always be found which remain unchanged after reflection from a corner cube, and if the corner cube is lossless these states correspond to linearly polarized fields.<sup>2</sup> Thus, it has been assumed in our calculations that the laser modes are in one of these polarization eigenstates, or that a decomposition of the fields into the polarization eigenstates has already been performed. Somewhat related polarization considerations have also arisen recently in connection with axicon-based resonators.<sup>5,6</sup>

#### IV. Experiment

A laser oscillator has been constructed to verify the performance of a corner cube resonator, and the experimental setup is sketched in Fig. 4. The corner cube is expected to have losses caused by light scatter at the apex and three edges, so a resonator with a large spot size has been used to minimize the effects of these scattering regions. For the configuration shown in Fig. 4 the large spot size is obtained by means of a beam expander within the cavity. The lenses required for these additional components result in cavity losses which must be overcome by a moderately high-gain laser medium. For these experiments pulsed argon has been used as opposed to other high-gain media simply because small argon plasma tubes, fill systems, and power supplies are readily available in the laboratory.

We now describe the setup in greater detail. The resonator consists of a 95% reflecting concave spherical mirror with a 1.2-m radius of curvature, a 2-mm diam plasma tube with a 50-cm length fitted with Brewster windows, a -2.7-cm focal length concave lens used together with a 23-cm focal length convex lens to form a beam expander, and finally the corner cube. This arrangement yields a spot size of  $\sim 0.5$ -cm radius at the corner cube. A variable aperture is placed between the two lenses to control the mode selection, and all optical elements are mounted on gimbal mounts. The lenses and corner cube are also equipped with  $x$ - $y$  translators. An additional mirror mount holding a plane 95% reflecting mirror can be positioned between the plasma tube and concave lens.

Both lenses are of the surplus variety, and only the

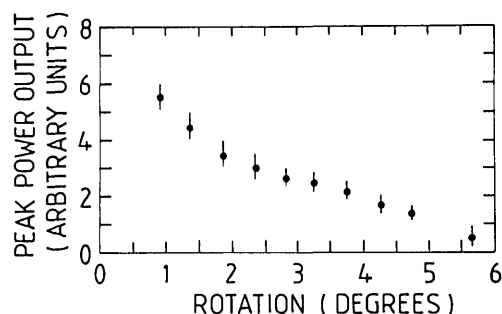


Fig. 5. Power output vs corner cube rotation.

convex lens is antireflection coated. The corner cube is a 2.5-cm diam element obtained from Pyramid Optics, Irvine, Calif. and is of the type used for surveying. Incident collimated light is reflected from the corner cube with  $<2$ -sec of arc deviation from parallelism, but the front face of the corner cube is not antireflection coated and the three triangular faces are not aluminized. The expected losses as a result of the beam expander and corner cube should then be at least 12% per pass, assuming that the AR coating on the convex lens yields no loss.

The plasma tube is equipped with indium electrodes of the type devised by Simmons and White.<sup>7</sup> These electrodes permit repeated exposure to the atmosphere without damage to the electrode material. The laser is maintained at a pressure of  $\sim 20$   $\mu$ m and is operated as a relaxation oscillator. The electrodes are connected to a 0.5- $\mu$ F storage capacitor charged to a few kilovolts which is discharged at a 120-Hz rate by ionizing the plasma with a small Tesla coil. This arrangement yields a laser output pulse of  $\sim 5$   $\mu$ sec in width with a 1-W peak power.

The laser bore, beam expander, and corner cube are first aligned with a small He-Ne laser by autocollimation. Then the plane and spherical mirrors are inserted into the cavity and aligned until laser action is initiated. Finally, the plane mirror is removed, and the beam expander is adjusted to initiate laser action with the corner cube. Laser threshold is somewhat sensitive to the position of the beam expander components, but in a practical application these components could be placed on a single rigid mount to maintain their relative positions. The laser is very insensitive to the orientation of the corner cube as indicated in Fig. 5. Indeed the laser can be made to operate satisfactorily by simply holding the corner cube by hand. Figure 5 shows typical data for the power output of the laser vs cube rotation as measured by a PIN diode. The dependence of power output on rotation is found to be essentially the same for both the  $x$  and  $y$  axes.

The resonator losses are experimentally determined by inserting additional loss into the corner cube resonator and the standard plane-concave resonator until laser action ceases. A comparison of the loss inserted into each resonator indicates that the loss for the beam expander and corner cube is slightly higher than the conservative 12% estimate mentioned here.

Typically several TEM modes are found to operate at the same time, so the variable aperture is used to-

gether with slight misalignments to limit the laser operation to a single transverse mode. The same modes can be observed with or without the plane mirror in the cavity, and the TEM<sub>00</sub>, TEM<sub>01</sub>, and TEM<sub>02</sub> Hermite-Gaussian modes can be readily obtained in this way. This confirms that the familiar Hermite-Gaussian modes are indeed solutions of the corner cube resonator. Laguerre-Gaussian modes can be represented as linear superpositions of Hermite-Gaussian modes, and as expected low-order Laguerre-Gaussian modes are also observed with our setup.

## V. Conclusion

We have solved the self-consistent Fresnel-Kirchhoff integral equation for the field distribution of the resonant modes of a resonator with a tilted corner cube mirror and a spherical mirror with infinite Fresnel number. Even when the inclination angle of the corner cube is quite large, low loss modes still exist and can be expressed in terms of the familiar Hermite-Gaussian functions. These modes have been demonstrated in experiments using Ar-ion lasers. The corner cube acts like a convex lens with radius of curvature  $-L/\tan^2\theta$  in the inclination direction and like a flat plane in the other direction. Resonators of this type might find application in adverse environments where maintenance of laser alignment is otherwise difficult.

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*Note added in proof:* Several additional references should be added here and in relation to ref. 1. Resonators with crossed 90° roof reflectors were studied by G. Gould, S. Jacobs, P. Rabinowitz, and T. Shultz, *Appl. Opt.* **1**, 533 (1962). The properties of roof resonators have also been investigated by L. Ronchi, *Appl. Opt.* **12**, 93 (1973); F. Pasqualetti and L. Ronchi, *J. Opt. Soc. Am.* **65**, 649 (1975); A. Hardy, F. Pasqualetti, and L. Ronchi, *Optica Acta* **22**, 761 (1975). The matrix for a retroreflector has been mentioned by A. Gerrard and J. M. Burch, *Matrix Methods in Optics* (Wiley, New York, 1975), p. 103. The diffraction pattern of a single corner cube has been studied by R. F. Chang, D. G. Currie, C. O. Alley, and M. E. Pittman, *J. Opt. Soc. Am.* **61**, 431 (1971); and a corner cube has recently been employed with a large volume CO<sub>2</sub> laser by R. A. Chodsko, et al., *Proceedings of the International Conference on Lasers* **81**, December 13-18 1981, New Orleans, La.

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